

Methods of Solving Stereometry Problems Using Geogebra Software

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Received: 17.09.2024

Revised: 19.10.2024

Accepted: 09.11.2024

ABSTRACT

Among modern accessible digital tools that allow constructing and transforming geometric objects interactively, the GeoGebra environment is in increasing demand among teachers and researchers. However, analyzing printed and electronic sources, we can conclude that the issue of using GeoGebra in solving positional stereometric problems is not fully addressed. The features of solving positional stereometric problems using the GeoGebra 5.0 environment in geometry lessons in high school and when teaching image methods to future mathematics teachers at a pedagogical university are revealed. Emphasis is placed on the advantages of using GeoGebra in teaching the construction of spatial figures and their sections compared to traditional teaching.

Keywords: GeoGebra, visualization of geometric objects, methods of teaching geometry, development of spatial thinking.

INTRODUCTION

The role of geometry in the formation and development of a full-fledged personality cannot be overestimated. Undoubtedly, it contributes to the understanding of the material and spiritual world; without geometry, successful work is unthinkable for a huge number of professions. School graduates aiming to enter universities choose a profile level when passing the unified state exam in mathematics. However, few of them begin to solve geometric problems, including tasks with a detailed answer.

MATERIALS AND METHODS

Accompanying theoretical material with interactive tasks that require constructing spatial figures and images on them helps to overcome these difficulties. Modern digital tools make it possible to construct and transform geometric 3D models interactively. Among them "Living Geometry", "Maple", "3D Sec Builder", "Mathematica", "GeoGebra".

In terms of accessibility and ease of use, GeoGebra certainly has an advantage. It is a dynamic, open-source math program that combines geometry, algebra, and calculus. It was developed by Markus Hohenwarter and the international community of programmers for learning and teaching mathematics. On the official website of the program [2] you can download various versions for popular operating systems, including mobile ones. Please note that it is possible to use the program and its individual components online.

The fact that GeoGebra has broad didactic potential in teaching geometry is confirmed by the publications of currently relevant educational and teaching aids [3, 4]. In particular, it can be used in educational and research activities of students, for example, when checking the correctness of formulated hypotheses [5]. Over the past decade, more and more articles have appeared related to the description of the program's capabilities in solving specific problems from various fields of mathematics: continuous drawing of a graph of a given function and its derivative, graphs of polynomials, animated images of second-order curves [6]; solving some algebraic problems using the animation-geometric method; solving visualized problems in algebra and planimetry; transformation of objects on a plane; solving stereometric problems, animated images of conic sections, etc.

RESULTS AND DISCUSSION

The main tools of the GeoGebra environment, necessary for constructing volumetric bodies and solving stereometric problems, are given in the manual by V. A. Smirnov and I. M. Smirnova [5]. However, the question of the advantages of using GeoGebra in solving positional stereometric problems in comparison with traditional teaching methods has not been fully disclosed.

Let us recall that the positional task is understood as the task of establishing the relative position of original figures from their images made in the same projection on the same plane [6, p. 39]. The classical positional problem is the problem of constructing sections of polyhedra and bodies of revolution by a plane.

The problem of constructing a section of a polyhedron by a plane comes down to two cases: constructing the intersection of a cutting plane with the faces of the polyhedron; constructing the intersection of a cutting plane with the edges of a polyhedron.

Note that the cutting plane is most often defined either by three non-collinear points, or by two different points and a direction, or by a point and two different directions. Let us consider the construction of a section of a pyramid by a plane defined by three non-collinear points.

Task. Given a quadrangular pyramid $SABCD$ and three points on its surface:

$P \in SA, G \in SC, F \in (SCD)$.

Construct: section (PGF).

The construction of spatial figures in the GeoGebra environment can be performed in various ways: by constructing this image manually using various geometric elements (points, straight lines, rays, segments, etc.) on the toolbar in the "Geometry" mode, or by selecting ready-made models of polyhedra and bodies of rotation in 3D graphics mode.

In our case, for example, you need to select the "Pyramid" icon, build the bottom base, and then specify the top. In this case, you can use the context menu by clicking the right mouse button to select functions such as setting axes, grid, selecting scale; sequential reproduction of the steps of constructing a polyhedron and its sections; you can change the color, size and character of lines and dots (Fig. 1). Let's mark the points belonging to the cutting plane using the "Point" tool and change their color for clarity.

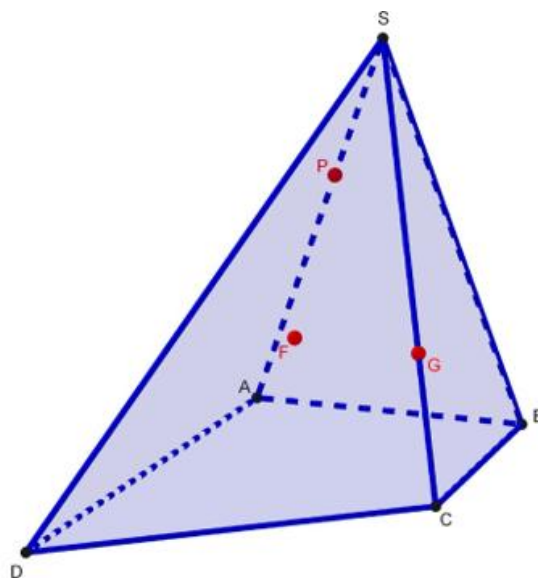


Fig 1. Image of a pyramid with given conditions

First way. Let's solve the problem using the trace method.

Analysis. Let us find the intersection of the cutting plane with the plane of the base of the pyramid: the trace of the cutting plane is straight line m , which is defined by two points: $M = PG \cap AC$, $N = PF \cap AK$, where K is the central projection of point F .

Construction. During construction we will use the "Straight" and "Intersection" tools.

1. $K : K = SF \cap CD$;
2. $M : M = PG \cap AC$;
3. $N : N = PF \cap AK$;
4. $m : M \in m, N \in m$;
5. $R : R = BC \cap m$;
6. $L : L = GR \cap SB$;
7. $E : E = GF \cap SD$;
8. Quadrangle $PLGE$ – the required section.

We visualize the constructed section using the “Polygon” tool and sequentially indicating its vertices. To make the image clearer, let’s set the colors of the objects by calling the context menu on the objects panel (Fig. 2).

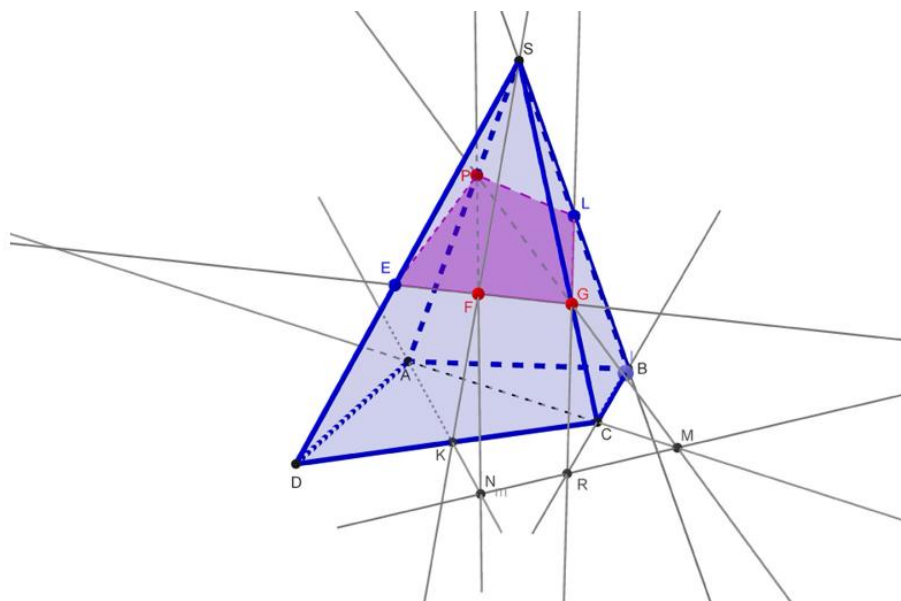


Figure 2. The desired section. Trace method

Note that with traditional teaching of the trace method, a situation may arise when the trace of the cutting plane is located outside the boundaries of the drawing, which makes further construction impossible. GeoGebra allows you to change the initial conditions of the problem, which eliminates this limitation. Thus, in the problem under consideration, you can move specified points on the edges and faces of the pyramid or change the shape of its base using the “Move” tool.

Second way. Let's solve the problem using the internal design method. To do this, we will hide the objects in the drawing obtained during construction using the first method by clicking on the dots near the corresponding elements in the objects panel.

Analysis. Let's find the intersection points of the cutting plane with the edges of the pyramid. P and G are the intersection points of the cutting plane with the edges SA and SC, respectively. Point E: = $GF \cap (BCD)$ – the point of intersection of the cutting plane with the edge SD (or its extension). To find the point of intersection of the cutting plane with the edge SB, it is necessary to find the line of intersection of the main and additional planes ($(SAC) \cap (BCD)$).

Construction

1. $O_1 : O_1 = AC \cap BD$;
2. $SO_1 : SO_1 = (SAC) \cap (BSD)$;
3. $O : O = SO_1 \cap PG$;
4. $EO \cap SB = L$;

5. PLGE – the desired section (Fig. 3).

The undoubted advantages of the GeoGebra program include the creation, without programming knowledge, of an animation drawing that clearly shows not only a figure in dynamics (movement), but also the possibility of changing it. The creation of animated drawings from various fields of mathematics in the GeoGebra environment is described in sufficient detail, for example, in the manual by S. V. Larin [1]. However, the manual does not consider dynamic models of positional tasks.

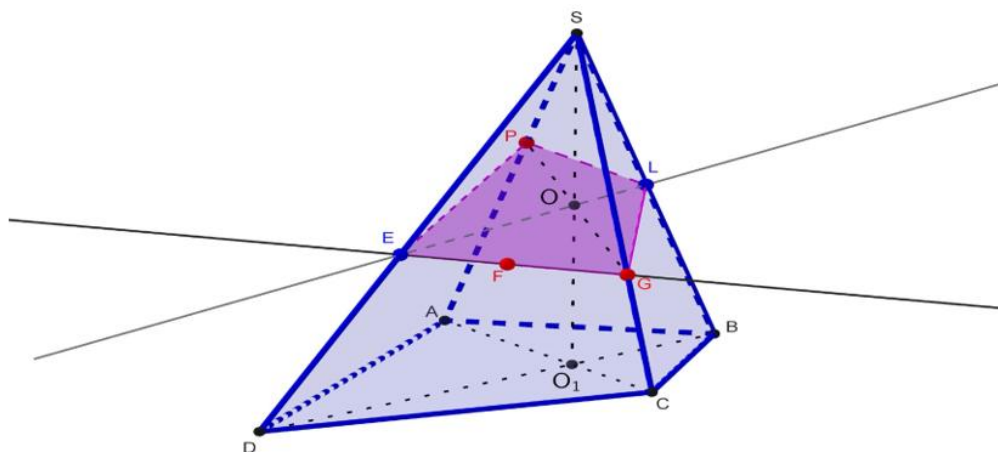


Figure 3. The required section. Internal Design Method

Depending on the type of polyhedron and the location of the cutting plane, there may be different polygons in the section. The number of sides (vertices) of a polygon cannot exceed the number of faces of the polyhedron. In the problem under consideration, the sections can be pentagons, quadrangles and triangles, since a quadrangular pyramid has five faces. Working in the GeoGebra environment, you can clearly illustrate its various sections on one image of a polyhedron (Fig. 4).

For example, a triangle in the section of a given pyramid SABCD can be obtained if, instead of point F, we take point J on the diagonal AC, and a pentagon can be obtained by replacing point F with point Q on edge CD.

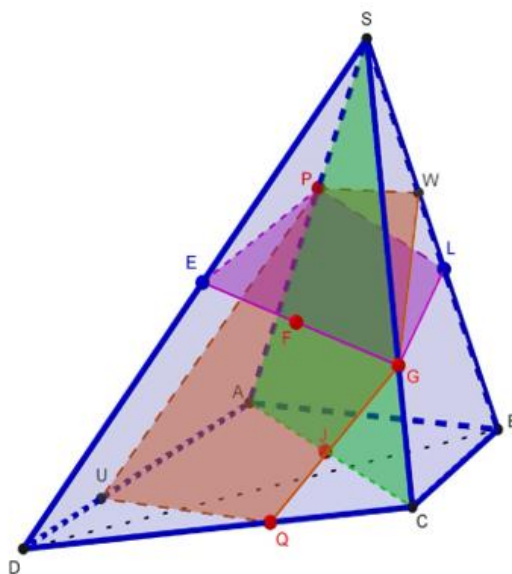


Figure 4. Demonstration of various sections in one drawing

CONCLUSION

By right-clicking on the construction canvas and selecting “Construction steps” in the context menu, you can call up a menu for controlling the animation of constructed sections.

Thus, the use of GeoGebra when teaching positional tasks has a number of advantages compared to traditional teaching, namely:

- the “Rotate” tool allows you to change the viewing angle of the polyhedron to meet the requirement of clarity for images of figures;
- in the process of solving one problem using the trace method, you can change the relative position of the elements of the constructed image of the polyhedron so that the trace of the cutting plane is conveniently located within the visibility of the projection drawing, thereby not requiring additional time costs for constructing a new polyhedron;
- by changing the positions of the elements of the desired section specified in the problem statement, you can see its different types in dynamics.

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