

# Role of magnetic field on blood flow with suspended silver nano particles through stenosed artery

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## ABSTRACT

The study of magnetic effects on silver nanoparticles suspended in the base fluid passing through an artery with overlapping stenosis is the focus of this publication. The study of magnetic effects on silver nanoparticles suspended in the base fluid passing through an artery with overlapping stenosis is the focus of this publication. A blood vessel with minor stenosis approximations is the subject of the study. Exact solutions for temperature, velocity, and flow resistance are found by solving the governing equations. The graphs have been used to study the effects of different parameters on the flow. According to the data taken into consideration, silver nanoparticles are effective at lowering the hemodynamics of stenosis and may have uses in biomedicine. According to the results, nanoparticles can be used as medicine carriers to reduce the negative effects of blood flow resistance.

**Keywords:** Magnetic field, overlapping stenosis, nanoparticles.

## INTRODUCTION

Heart disease is a term used to delineate many different parts affecting the heart. The coronary heart disease is a familiar type of the heart disease. This heart disease is the most significant root of death worldwide today and this death rate is still on the increase and it is expected more in coming and main reason for all heart diseases is hardening of the arteries. This hardening is due to deposits of fatty substances on the inside of the artery. In this condition arteries are unable to maintain regular flow. The other name for the hardening of the arteries is stenosis or arteriosclerosis. Suppose, the arteries of the heart are affected by stenosis. It results abnormal cardiac rhythms, heart failure and heart attacks etc.

In the view of this, study of blood flows in a stenosed arteries play a very important role to examine the different types of heart diseases. Based on this, several investigators are studied the blood flows by taking blood as Newtonian or Non-Newtonian according to shear rates [1-4].

Stenosis may form in series like multiple or overlapping or irregular shapes. A mathematical model of non-Newtonian blood flow in a tapered overlapping constriction is investigated by Ismail et al., [5]. Srivastava et al. [6] studied about the Non-Newtonian effects on blood flows in an overlapping stenosis. Gopal Chandra et al., [7] developed a numerical model of the pulsatile blood flow in a porous overlapping abnormality under influence of magnetic fields.

Nanotechnology plays a prominent role in the branch fluid dynamics. It has huge applications in our everyday activities like medicines, chemicals, food, automobiles and much more. It also a powerful tool into the forthcoming of medicine and biomechanics. A fluid consists nano scaled particles is known as nanoparticles. Non-Newtonian nanofluids are extensively applied in several biological appliances like protein detection, probing of DNA structure and cancer therapy etc. Nanofluids was originated by Choi [8]. Additional related works can be seen over [9-13]. A.S Dawood et al. "An analytical investigation of the multi-effect analysis of nanofluid flow in stenosed arteries with a changing pressure gradient 5(2023):382 in N Applied Sciences.[14]. Sushila and Prasun Choudhary. "Effects of an unsteady MHD hybrid nanofluid with gyrotatic microorganisms and thermal radiation on a non-linear stretchable porous sheet." Thermo Fluids International Journal 23 (2024): 100788 [15]

Interested by the above readings, an effort has been made in this paper to examine the effects overlapping stenosis of has been explored under the mild stenosis hypothesis. The examination is done analytically. The influence of diverse significant parameters on flow variables has been detected over the diagrams.

### Mathematical Formulation

Consider the steady, incompressible and axisymmetric flow of blood through a circular tube of length 'L', in the presence of overlapping stenosis as shown in Fig.1.

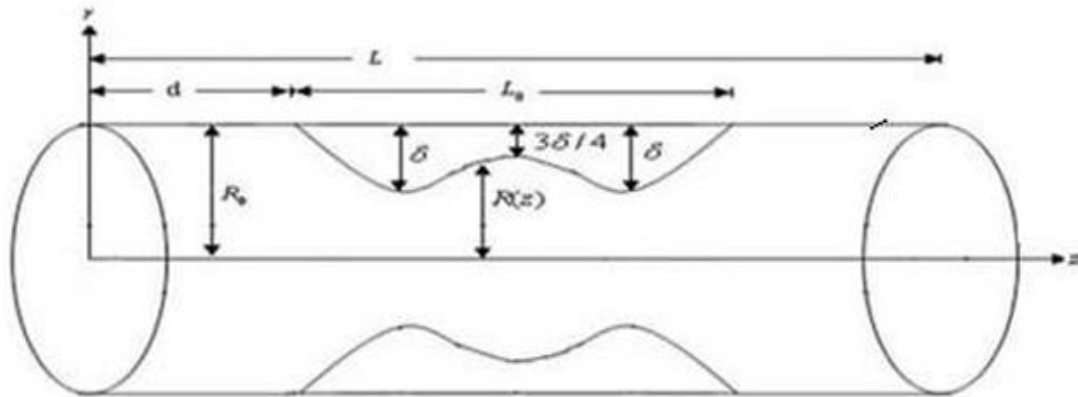


Fig 1: Geometry of the Stenosed artery

The geometry of the artery wall is defined mathematically as [6]

$$h = \frac{R(z)}{R_0(z)} = 1 - \frac{3\delta}{2R_0L_0^4} [11(z-d)L_0^3 - 47(z-d)^2L_0^2 + 72(z-d)^3L_0 - 36(z-d)^4],$$

$$d \leq z \leq d + L_0,$$

$$= 1, \text{ otherwise.} \quad (1)$$

Here,  $d$  is the position of the stenosis,  $L_0$  is its length,  $\delta$  is its maximum height, and  $z=d+L_0/6, z=d+5L_0/6$  are its locations.  $R(z)$  is the tube radius in the stenotic zone, and  $R_0(z)$  is the radius in the non-stenotic section. The essential height from the origin is determined to be  $3\delta/4$  at  $z=d+L_0/2$ .

For an incompressible nanofluid, the governing equations for mass, momentum, and temperature conservation are as follows:  $\partial v/\partial r + v/r + \partial u/\partial z = 0$  (see ref[11])

$$\frac{\partial v}{\partial r} + \frac{v}{r} + \frac{\partial u}{\partial z} = 0 \quad (2)$$

$$\rho_{nf} \left( v \frac{\partial v}{\partial r} + u \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial r} + \mu_{nf} \frac{\partial}{\partial r} \left( 2 \frac{\partial v}{\partial r} \right) + \mu_{nf} \frac{\partial}{\partial z} \left( 2 \frac{\partial v}{\partial z} + \frac{\partial u}{\partial r} \right) \quad (3)$$

$$\rho_{nf} \left( v \frac{\partial u}{\partial r} + u \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu_{nf} \frac{\partial}{\partial z} \left( 2 \frac{\partial u}{\partial z} \right) + \frac{\mu_{nf}}{r} \frac{\partial}{\partial r} \left[ r \left( \frac{\partial v}{\partial z} + \frac{\partial u}{\partial r} \right) \right] - g\rho_{nf} \alpha (T - T_0) - \sigma B_0^2 u \quad (4)$$

$$\left( v \frac{\partial T}{\partial r} + u \frac{\partial T}{\partial z} \right) = \frac{K_{nf}}{(\rho c_p)_{nf}} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{Q_0}{(\rho c_p)_{nf}} \quad (5)$$

With the conditions

$$\frac{\partial u}{\partial r} = 0, \frac{\partial T}{\partial r} = 0 \text{ at } r = 0 \quad (6)$$

$$u = 0, T = 0 \text{ at } r = h \quad (7)$$

$T$  is the fluid's temperature,  $Q_0$  is the constant heat absorption or heat generation,  $\mu_{nf}$  is the dynamic viscosity,  $\rho_{nf}$  is the density,  $k_{nf}$  is the thermal conductivity,  $\alpha_{nf}$  is the thermal diffusivity, and  $(\rho c_p)_{nf}$  is the heat capacitance of the nanofluid, as shown in the equations above.

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}} \cdot k_{nf} = k_f \left\{ \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + 2\varphi(k_f - k_s)} \right\}$$

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_p, (\rho c_p)_{nf} = (1-\varphi)(\rho c_p)_f + \varphi(\rho c_p)_p$$

Introducing the following non-dimensional variables

$$\bar{r} = \frac{r}{R_0}, \quad \bar{z} = \frac{z}{L_0}, \quad \bar{v} = \frac{L_0}{\delta U} v, \quad \bar{u} = \frac{u}{U}, \quad \bar{d} = \frac{d}{L_0}, \quad R = \frac{R}{R_0}$$

$$M^2 = \frac{\sigma B_0^2 R_0^2}{\mu_f}, G_r = \frac{g \alpha R_0^2 T_0 \rho_{nf}}{U \mu_f}, \delta = \frac{\delta}{R_0}, \theta = \frac{T - T_0}{T_0}$$

$$\bar{p} = \frac{U L_0 \mu}{R_0^2} p, \quad \beta = \frac{Q_0 R_0^2}{k_f T_0},$$

where U is the average velocity over the R<sub>0</sub> segment of the tube.

After using the non-dimensional variables in equations (2)-(5) and also making use of the mild stenosis

conditions  $\epsilon = \frac{R_0}{L_0} = o(1), \frac{\delta}{R_0} \ll 1$  the reduced equations along with the boundary conditions are (after dropping the bars)

$$\frac{\partial p}{\partial r} = 0 \quad (8)$$

$$\frac{dp}{dz} = \frac{1}{(1-\varphi)^{2.5}} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) - M^2 u + G_r \frac{\theta}{r} \quad (9)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right) + \beta \left( \frac{k_{nf}}{k_f} \right) = 0 \quad (10)$$

The Hartmann number, heat absorption parameter, and Grashof number are denoted by M, β, and G<sub>r</sub>, respectively.

The following are the no-boundary conditions:

$$\frac{\partial u}{\partial r} = 0, \frac{\partial \theta}{\partial r} = 0 \quad \text{at } r = 0 \quad (11)$$

$$u = 0, \theta = 0 \quad \text{at } r = h \quad (12)$$

### Solution

The exact solutions of equations (9) and (10) are obtained as

$$\text{Velocity, } u = \left\{ \frac{M^2 \frac{dp}{dz} - G_r \beta \left( \frac{k_f}{k_{nf}} \right)}{I_0(Mh\sqrt{(1-\varphi)^{2.5}})} \right\} I_0(Mr\sqrt{(1-\varphi)^{2.5}}) - \frac{G_r \beta}{4M^2} \left( \frac{k_f}{k_{nf}} \right) (r^2 - h^2) - \frac{1}{M^2} \frac{dp}{dz} + \frac{G_r \beta}{M^2} \left( \frac{k_f}{k_{nf}} \right) \quad (13)$$

$$\text{Temperature, } \theta = \frac{-\beta}{4} \left( \frac{k_f}{k_{nf}} \right) (r^2 - h^2) \quad (14)$$

The dimension less flux q<sub>is</sub>

$$q = 2 \int_0^r r u dr.$$

$$= \frac{1}{M^2} \frac{dp}{dz} - \frac{Gr\beta}{M^2} \left( \frac{k_f}{k_{nf}} \right) \left( \frac{h I_1(Mh\sqrt{(1-\varphi)^{2.5}})}{M\sqrt{(1-\varphi)^{2.5}}} \right) + \frac{Gr\beta}{16M^2} \left( \frac{k_f}{k_{nf}} \right) + \frac{Gr\beta h^2}{2M^2} \left( \frac{k_f}{k_{nf}} \right) - \frac{h^2}{M^2} \frac{dp}{dz}$$

It implies,

$$\frac{dp}{dz} = \frac{q - \frac{Gr\beta}{16M^2} \left( \frac{k_f}{k_{nf}} \right) - \frac{Gr\beta h^2}{2M^2} \left( \frac{k_f}{k_{nf}} \right) + \frac{Gr\beta}{M^2} \left( \frac{k_f}{k_{nf}} \right) \left( \frac{h I_1(Mh\sqrt{(1-\varphi)^{2.5}})}{I_0(Mh\sqrt{(1-\varphi)^{2.5}})} \right)}{\frac{-h^2}{2M^2} + \frac{h}{M^3} \left( \frac{I_1(Mh\sqrt{(1-\varphi)^{2.5}})}{I_0(Mh\sqrt{(1-\varphi)^{2.5}})} \right)} \quad (15)$$

The definition of the impedance resistance  $\lambda$  is  $\lambda = \frac{\Delta p}{Q} = - \int_0^1 \frac{dp}{dz} dz$

The definition of the pressure drop without stenosis ( $h=1$ ) is as  $\Delta p_n = \left[ - \int_0^1 \frac{dp}{dz} dz \right]_{h=1}$

$\lambda_n = \frac{\Delta p_n}{Q}$  is the definition of the impedance resistance in a normal artery.

The formula for the normalized impedance is  $\tilde{\lambda} = \frac{\lambda}{\lambda_n}$

Additionally, the definition of the wall shear stress  $\tau_h$  is  $\tau_h = - \frac{h}{2} \frac{dp}{dz}$

The impedance resistance  $\lambda$  is defined as  $\lambda = \frac{\Delta p}{Q} = - \int_0^1 \frac{dp}{dz} dz$

The pressure drop without stenosis ( $h = 1$ ) is defined as  $\Delta p_n = \left[ - \int_0^1 \frac{dp}{dz} dz \right]_{h=1}$

The impedance resistance in the normal artery is defined as  $\lambda_n = \frac{\Delta p_n}{Q}$

The normalized impedance defined as  $\tilde{\lambda} = \frac{\lambda}{\lambda_n}$

And the wall shear stress  $\tau_h$  is defined as  $\tau_h = - \frac{h}{2} \frac{dp}{dz}$

## RESULTS AND DISCUSSION

The graphs in this section are used to interpret the results. Figs. 2–3 illustrate how temperature is affected by stenosis height ( $\delta$ ) and heat source or sink factors ( $\beta$ ). As the stenosis height and heat absorption parameter increase, the temperature also rises. The tube's center has the highest temperature, while the areas closest to the walls have the lowest. Additionally, it is observed that Ag blood has a greater temperature variance than pure blood.

Figs. 4–7 show the velocity change versus the radial axis for various values of  $\delta$ ,  $\beta$ ,  $G_r$ , and  $M$ . The velocity is found to be zero at the artery wall and to be high in the middle of the tube before gradually decreasing. Additionally, it is observed that the velocity rises with the Grashof number, heat absorption parameter, and stenosis height but falls with the Hartman number. The velocity of the Ag-concentrated fluid is higher than that of the pure fluid ( $\phi=0$ ).

The change in impedance resistance with relation to stenosis height is shown in Figs. 8–11 for a range of heat absorption parameter, Grashof number, Hartman number, and flow rate values. It is observed that the impedance

resistance falls as the heat absorption parameter, Grashof number, rises and rises with the height of the stenosis, Hartman number, and flow rate. When comparing Ag blood to pure blood, we saw that the flow resistance was lower in each of these figures.

Figures 12-15 exhibit the wall shear stress against the axial distance for various values of the stenosis height, Grashof number, Hartman number, and heat absorption parameter  $\beta$ . It is observed that the wall shear stress decreases with Grashof and Hartman numbers and increases with stenosis height and heat absorption parameter. Additionally, it is observed that the wall shear stress rises from  $z=0$  to  $z=0.26$ , then falls to  $z=0.4$  before rising once again to  $z=0.53$  from there it gradually decreases and approaches zero. This shows that the walls shear stress is maximum at the throats and minimum at the critical height of the stenosis. It is keenly observed here that the wall shear stress is more for the fluid when the silver concentration is more in the fluid. For pure fluid the shear stress at the walls is low.

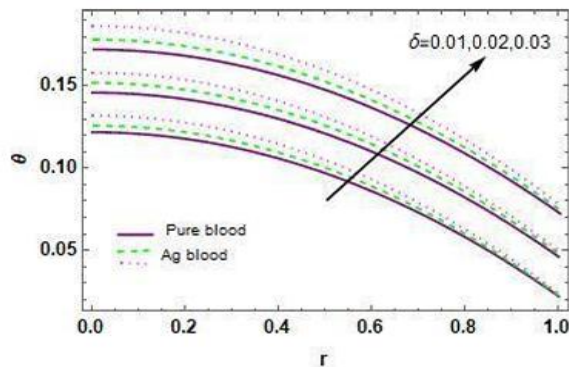


Fig 2: Change in temperature for different values of  $\delta$

$$d = 0.2, L_0 = 0.4, \beta = 0.1, L = 1, z = 0.1$$

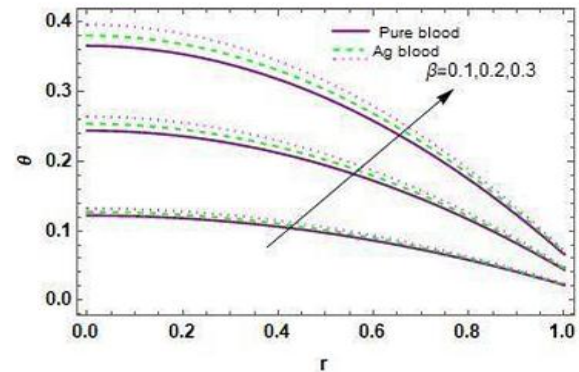


Fig 3: Change in temperature for different values of  $\beta$

$$d = 0.2, L_0 = 0.4, \delta = 0.01, L = 1, z = 0.1$$

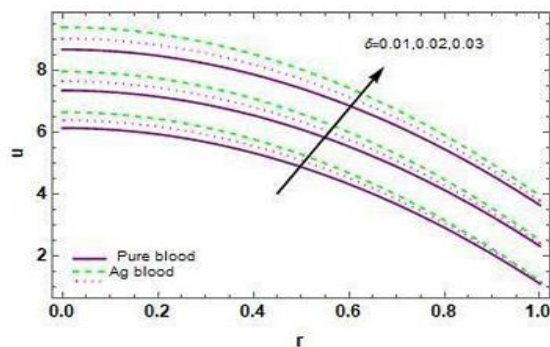


Fig 4: Change in velocity for different values of  $\delta$

$$d = 0.2, L_0 = 0.4, \beta = 0.1, L = 1, z = 0.1, Gr = 2; M = 0.1$$

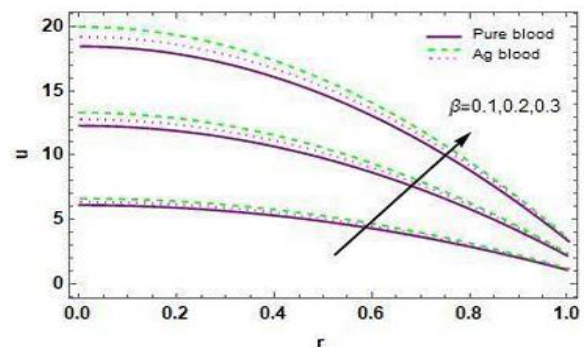


Fig 5: Change in velocity for different values of  $\beta$

$$d = 0.2, L_0 = 0.4, \delta = 0.01, L = 1, z = 0.1, Gr = 2; M = 0.1$$

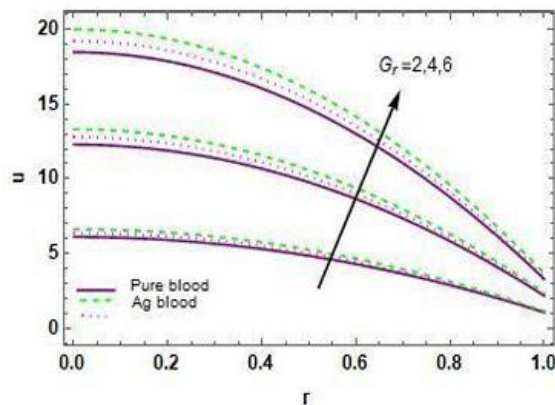


Fig 6: Change in velocity for different values of  $G_r$

$$d = 0.2, L_0 = 0.4, \beta = 0.1, L = 1, z = 0.1, \delta = 0.01, M = 0.1$$

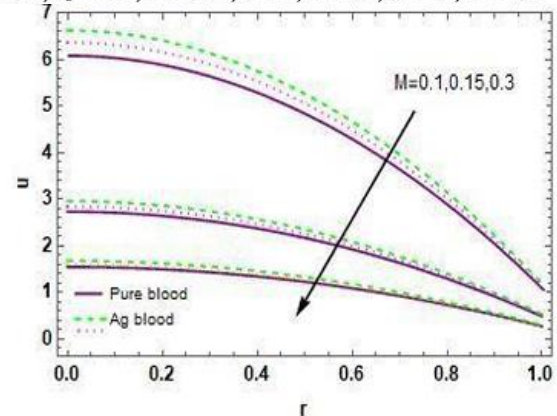


Fig 7: Change in velocity for different values of  $M$

$$d = 0.2, L_0 = 0.4, \delta = 0.01, L = 1, z = 0.1, Gr = 2, \beta = 0.1$$



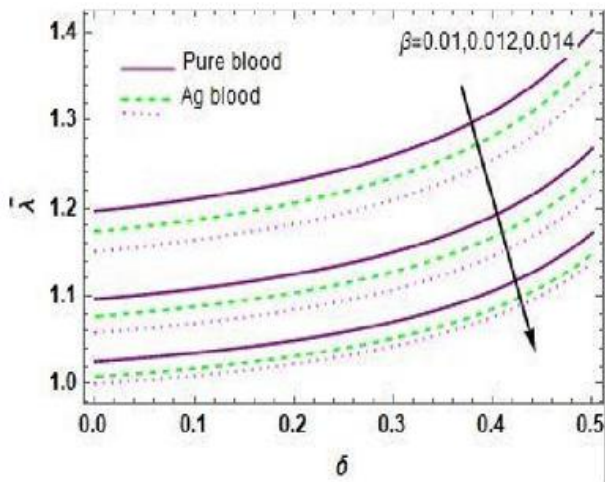


Fig 8: Change in flow resistance for different values of  $\beta$   
 $d = 0.2, L_0 = 0.4, L = 1, q = 0.01, Gr = 2, M = 0.1$

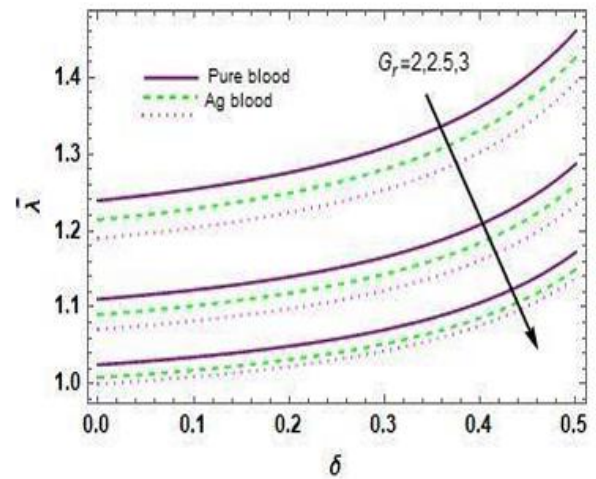


Fig 9: Change in flow resistance different values of  $Gr$   
 $d = 0.2, L_0 = 0.4, L = 1, q = 0.01, \beta = 0.1, M = 0.1$

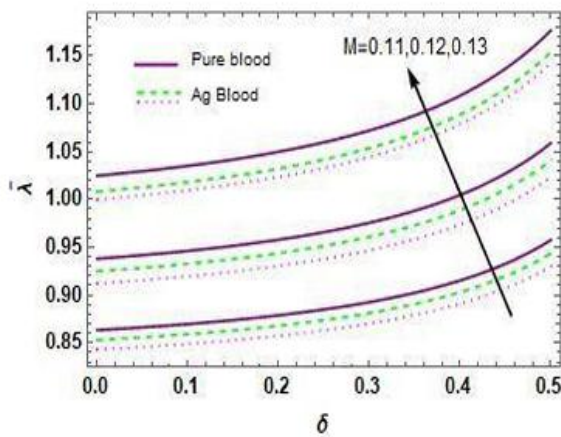


Fig 10: Change in flow resistance for different values of  $M$   
 $d = 0.2, L_0 = 0.4, L = 1, q = 0.01, \beta = 0.1, Gr = 2$

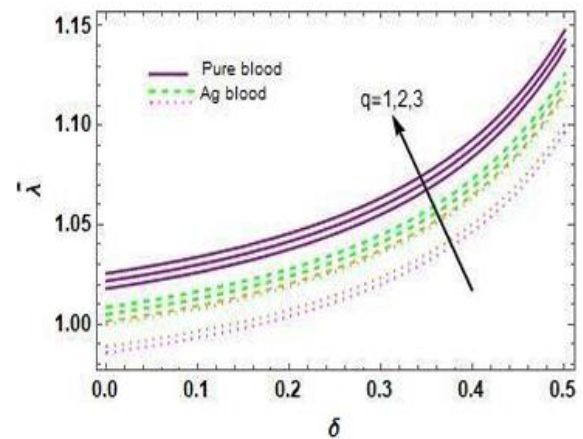


Fig 11: Change in flow resistance different values of  $q$   
 $d = 0.2, L_0 = 0.4, L = 1, \beta = 0.1, M = 0.1$

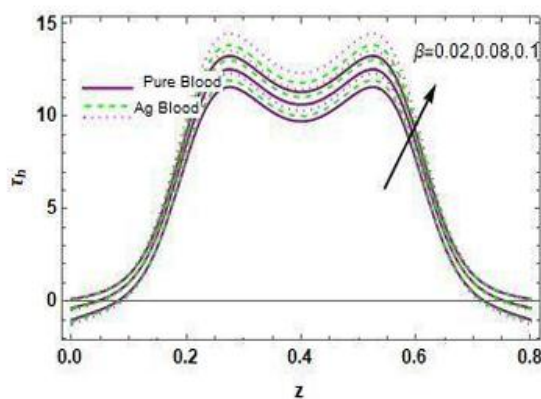


Fig 12: Wall shear stress for different values of  $\beta$   
 $0.2, L_0 = 0.4, L = 1, q = 1, \delta = 0.1, M = 1, Gr = 2$

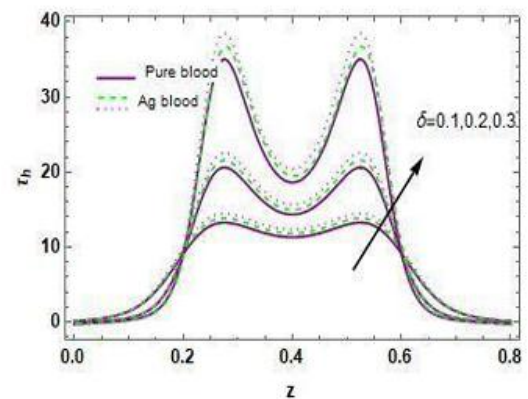
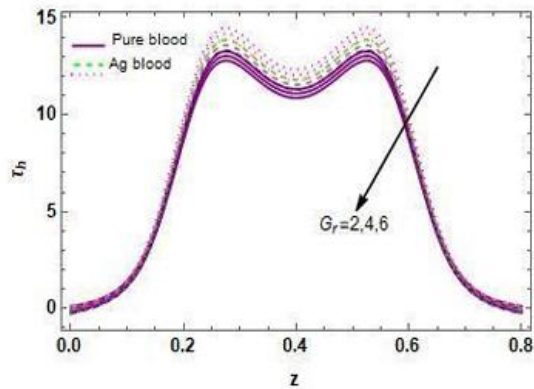
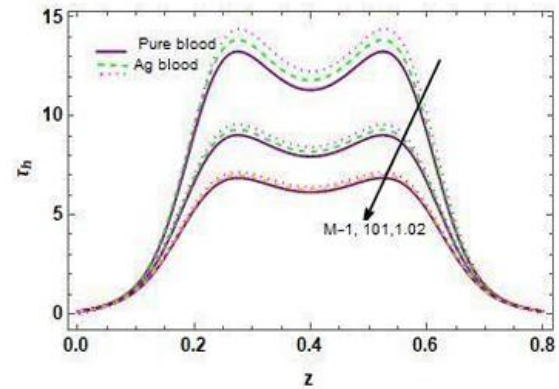


Fig 13: Wall shear stress for different values of  $\delta$   
 $d = 0.2, L_0 = 0.4, L = 1, q = 1, \beta = 0.02, M = 1, Gr = 2$

Fig 14: Wall shear stress for different values of  $G_r$ 

$$d = 0.2, L_0 = 0.4, L = 1, q = 1, \delta = 0.1, M = 1, \beta = 0.02$$

Fig 15: Wall shear stress for different values of  $M$ 

$$d = 0.2, L_0 = 0.4, L = 1, q = 1, \beta = 0.02, \delta = 0.1, Gr = 2$$

## CONCLUSION

The present analysis is concerned to the silver nanoparticles in the base fluid for the axisymmetric flow through an artery with overlapping stenosis. In this model we studied the effects of various parameters like stenosis height, Grashof number, heat source or sink parameter, Hartman number on the flow characteristics velocity, resistance to the flow and wall shear stress.

The main conclusions are given below:

1. The velocity profile increases with height of stenosis, heat absorption parameter, and Grashof number but decreases with Hartman number.
2. It seen that the impedance resistance increases with the height of the stenosis, Hartman number, flow rate and decreases as heat absorption parameter, Grashof number increases.
3. It is observed that the wall shear stress increases with stenosis height and heat absorption parameter and decreases with Grashof number and Hartman number.

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